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SIMULATION OF HEAT EXCHANGE OF A LITHOSPHERIC PLATFORM IN THE ZONE OF SUBDUCTION. I. STATEMENT OF THE PROBLEM

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The problems of numerical simulation of the processes of the thrust of an oceanic platform under a continental platform are considered. Numerical experiments on the calculation of the thermal state and evolution of subsidence of the oceanic platform in the zone of subduction are carried out. It is found that the maximum depth of subsidence of the oceanic platform does not exceed 720 km.

In recent times, an ever increasing number of research workers attempting to explain the nature of the mechanism of convection in the earth's mantle have directed their attention to the zone of subduction, because the mechanism involved in subsidence of the subduction platform into the mantle is an essential part of the mechanism of mantle convection.

Subduction zones (island arcs and active continental margins) of the Pacific Ocean type are characterized by intense seismicity. A considerable portion of the seismic activity is concentrated in the region of an inclined plane subsiding at an angle of about 45^0 inward from the trench under the island arc or continental margin. These seismic planes (Benjoff zones) represent large tectonic displacements. Most often, the Benjoff zones subside at an angle of 45^0, but in different island arcs a range of subsidence angles of from 30^0 to 90^0 was discovered. Even in the same island arc the subsidence angle of the zone can change appreciably, usually increasing with an increase in depth.

In a first approximation, the subsided part of a subducted platform can be considered a very viscous fluid ($\mu = 10^{23}$ Pa·sec) [1, 2]. According to [1, 2], the viscosity of the mantle underlying the lithosphere is assumed to be $\mu = 10^{20}$ Pa·sec.

From the results of [3, 4] it follows that the decay of radioactive elements contained in the earth's crust and mantle makes a substantial contribution to the energy balance of the earth. Therefore, when stating a mathematical model, it is necessary to take into account the heat release of radioactive elements as a result of their decay.

In the works published for the past 20 years and devoted to the development and use of quantitative methods for investigating the zones of subduction (underthrust) of lithospheric platforms, thermomechanical convection is predominantly simulated and the stressed state is determined (see, for example, [2, 5-7]).

The problem of determining the geometric form of the subsided portion of a subducted platform by quantitative methods remains virtually open. In [1] a model is suggested that explains the mechanism of underthrust from the viewpoint of the gravitational instability of the heavy lithosphere above the lighter asthenosphere. The geometric form of the subsided portion of a subducted platform obtained in [5] is disputable, since at the present time there are no reliable proofs supporting the existence of an abnormal mantle.

Investigation of the olivine–spinel phase transitions and calculation of the stressed state of the subsided portion of a subducted platform are given in [8]. In that work the olivine–spinel phase transition is taken into account as a density jump in the zone of transition. A mathematical model developed by the authors of [9] makes it possible to determine the depth of subsidence from the known trajectory and rate of subsidence of the platform.

Two-dimensional models of convection in a compressible liquid with constant and variable viscosities presented in [3] showed that in the case of convection with a variable viscosity, the nonlinear interaction of

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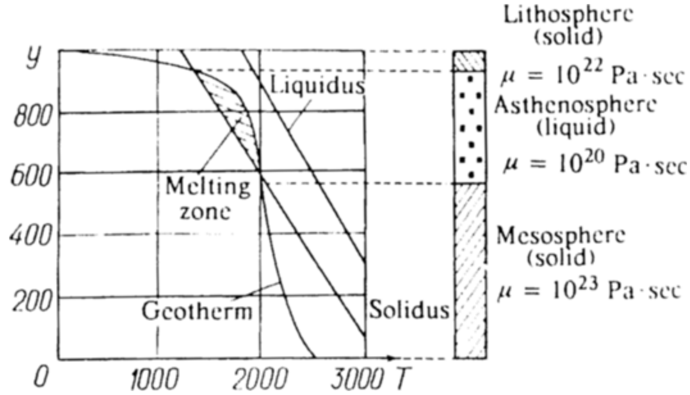

Fig. 1. Diagram of the solid body–liquid phase transition. y, km; T, K..

compression, adiabatic and viscous heating, temperature-dependent rheological properties, pressure, and shear stress leads to important consequences: the ascending flow expands and plumes are retained, the flow is strongly concentrated as the subsidence region is approached and loses coupling with the inner regions of the cell, separating from them by a zone of reduced viscosity. Around the submerging platform two regions of reduced viscosity are formed, which effectively prevent mixing of the platform material with the surrounding mantle.

According to seismic investigations, the material of the mantle under the continental and oceanic platforms is in a molten state to depths of up to 500 km.

In [1] a mathematical model of the simultaneous motion of a melt and a solid phase (partial melting) is suggested. It is shown that under the action of deformation such a system stratifies, i.e., a system of alternating bands depleted and enriched in the molten substance is formed.

Melting of the mantle material occurs over a rather large range (from the solidus tempearture T_s to the liquidus temperature T_l) and is accompanied by the absorption of heat L_F (latent heat of melting). Curves of phase-transition temperature versus depth that characterize the solid body–liquid transition diagram are presented in Fig. 1.

If we assume that the material of the lithosphere and of the underlying mantle is a viscoelastic fluid, then, using methods of numerical simulation, it is possible to describe the zone of subduction by a mathematical model whose solution will allow us to obtain the temperature and velocity fields and to follow the evolution of the melting front. In this connection, of great interest is determination of the trajectory of the melting front of the oceanic platform during its subsidence into the earth's mantle. This will make it possible to estimate the effect of the subsidence of the oceanic platform on the processes occurring in the Benjoff zone. In [2] a simplified model was considered which involved Newtonian rheology and constant solidus and liquidus temperatures of the mantle pyrolite. In the work considered, non-Newtonian rheology is introduced into the mathematical model; to calculate the zones of partial melting, solidus and liquidus temperatures were used that depended on the water pressure and concentration.

To develop a mathematical model for the subduction zone, the following assumptions were adopted:

1. The material of which the lithosphere and underlying mantle consist is considered a viscous incompressible fluid.

2. The interface between the lithosphere and the mantle is the isotherm of the solidus temperature of peridotite T_s.

3. The viscosity, thermal conductivity, and heat release of the material are determined as functions of the position of the phase interface between the lithosphere and mantle in the following manner:

$$\{\mu, \lambda, q_v\} = \begin{cases} \{\mu_1, \lambda_1, q_{v1}\} & \text{for} \quad T \le T_s; \\ \{\mu_2, \lambda_2, q_{v2}\} & \text{for} \quad T > T_s; \end{cases} \tag{1}$$

The subscript 1 refers to the characteristics of the lithosphere and subscript 2 to the characteristics of the earth's mantle.

Determination of the rheology of the lithosphere, asthenosphere, and mezosphere is a rather complex problem. Some approaches to the description of their rheology are presented in [4, 10, 11]. And though the existing opinions on the rheology of the lithosphere and of the earth's mantle are ambiguous, the most appropriate dependence for calculation of dynamic viscosity is the following:

$$\mu = \frac{kT}{1 + \frac{\tau^2}{\tau_t^2}} \exp\left[\frac{E^* + PV^*}{RT}\right], \tag{2}$$

where T is the temperature, E^* and V^* are the energy and volume of activation, R is the gas constant, τ is the shear stress, τ_t is the shear stress at which the dislocation mechanism of creep appears, and k is the Boltzmann constant.

The quantity τ_t is defined by the empirical formula

$$\tau_t = 4 \cdot 10^4 \left(\frac{T}{T_m} - 0.25\right)^{-4}, \quad (\mathrm{N/m^2}),$$

where T_m is the melting temperature of dry fosterite.

We shall adopt dry fosterite as the basic component of the mantle; then for a developed flow the quantities entering into formula (2) will have the following values:

$$T_m = 2170 + 1.5 \cdot 10^{-3} y,\ \mathrm{K};\quad V^* = 1.1 \cdot 10^{-5},\ \mathrm{m^3/kmole};\quad E^* = 5.225 \cdot 10^5\ \mathrm{J/mole};$$

pressure P is determined as the hydrostatic pressure: $P = P_{\mathrm{at}} + \rho g y$, $\mathrm{N/m^2}$.

When solving the Navier-Stokes equations of motion, the value of dynamic viscosity was determined from relation (2). Along with relation (2), we also considered Newtonian rheology, where the dynamic viscosity assumed the following values:

$$\mu = \begin{cases} A\mu_0, & T < T_s; \\ \mu_0, & T \geq T_s. \end{cases} \tag{3}$$

The use of expression (3) to determine viscosity makes it possible to model the asthenospheric layer as a region owing its existence to partial melting of the mantle. The coefficient of the drop in viscosity A changes within the limits of $1-1000$. To decrease the fluctuations of the solution, in calculations the dependence (3) was approximated by a smooth function

$$\mu = \mu_0 \,[1 + (A - 1)] \left(0.5 + \frac{1}{\pi} \tan \frac{T_s - T}{h}\right). \tag{4}$$

Calculations were also performed for constant (mean integral) values of μ.

4. The value of the matertial density depends on the position of the phase interphase between the lithosphere and mantle and is determined from the formulas

$$\rho = \begin{cases} \rho_1 (1 - \beta T) & \text{for} \quad T \leq T_s; \\ \rho_2 (1 - \beta T) & \text{for} \quad T > T_s; \end{cases} \tag{5}$$

5. According to investigations of the fractional melting of the mantle pyrolite carried out by Ringwood and Green, the relative quantity of the solid phase $\varphi = V_s/(V_l + V_s)$ (V_s, V_l are the volumes of the solid and liquid

TABLE 1. Physical Parameters

Parameters	Lithosphere	Mantle
Viscosity μ, (Pa·sec)	10^{22}	10^{20}
Thermal conductivity λ, W/(m·K)	3	5
Density ρ, kg/m^3	3000	3300
Heat release q_v, W/m^3	$-5\cdot 10^{-6}$	10^{-9}
Specific heat capacity c_p, J/(kg·K)	1200	1200
Coefficient of volumetric expansion β, K^{-1}	$3\cdot 10^{-5}$	$3\cdot 10^{-5}$
Heat of melting L_F, J/kg	$4\cdot 10^5$	$4\cdot 10^5$

phases) that characterizes the melting and crystallization of the material is a function of the temperature, pressure, and of the concentration of the water. In the calculations in the present work, we used the following relation [9]:

$$\psi = \begin{cases} 1 & \text{for } T < T_s ; \\ 2\left(\dfrac{T - T_s}{T - T_l}\right)^3 - 3\left(\dfrac{T - T_s}{T_l - T_s}\right)^2 + 1 & \text{for } T_s \le T \le T_l ; \\ 0 & \text{for } T > T_l . \end{cases} \tag{6}$$

Here T_s and T_l are functions of the water pressure and concentration. It is assumed that the upper and lower zones of partial melting are determined by the limiting value $\psi_{\lim} = 0.95$ for the mantle conditions, i.e., when $\psi \le \psi_{\lim}$ the mantle material is in a liquid state and when $\psi > \psi_{\lim}$ it is in a solid state.

The values of the physical quantities for the lithosphere and mantle are presented in Table 1.

According to the hypothesis advanced by G. Hess in 1960 [12], the motion of the lithospheric platforms from the zones of formation (middle oceanic crests) to the zones of underthrust (island arcs) is rectilinear, therefore the problem of subsidence of the lithospheric platform into the earth's mantle can be considered two-dimensional (the x axis is directed along the motion of the platform, the y axis is directed along the normal to the plane of the platform). The geometry of the problem is depicted in Fig. 2.

Within the framework of the assumptions made, the mathematical model of the problem considered in the present work is described by the following system of differential equations

motion equations of a solid medium (Navier-Stokes equations):

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(2\mu\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial x}\right), \tag{7}$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\left(\mu\frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(2\mu\frac{\partial v}{\partial y}\right) + \frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial y}\right) + \rho g\beta\,(T - T_1), \tag{8}$$

where T_1 is a certain fixed temperature (reference point);

continuity equation:

$$\frac{\partial\rho}{\partial t} + \frac{\partial\rho u}{\partial x} + \frac{\partial\rho v}{\partial y} = 0 ; \tag{9}$$

energy conservation equation:

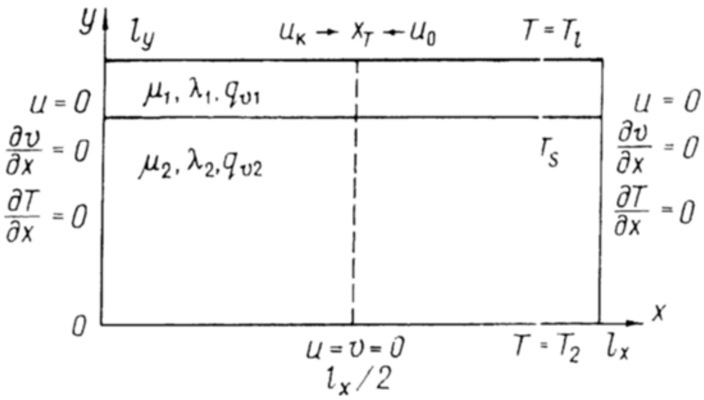

Fig. 2. Physical statement of the problem.

$$\rho c_{ef} \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial x} \left(\lambda \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(\lambda \frac{\partial T}{\partial y} \right) +$$

$$+ q_v + 2\mu \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 + \frac{1}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right] ; \quad (10)$$

where

$$c_{ef} \equiv \begin{cases} c_p & \text{for } T < T_s ; \\ c_p = L_F \dfrac{d\psi}{dt} & \text{for } T_s \le T \le T_l ; \\ c_p & \text{for } T > T_l ; \end{cases} \quad (11)$$

state equation:

$$\rho = \rho_i (1 - \beta T) , \quad i = 1 , 2 . \quad (12)$$

The model adopted employs the Boussinesq approximation.

We seek a solution of the problem in a rectangular region with dimensions l_x along the x axis and l_y along the y axis (Fig. 2) in which two lithospheric platforms move opposite each other. Underlying the platforms is the mantle.

The upper boundary of the computational domain is assumed to be the day surface of the earth. At a depth of about 100 m from the earth's surface the temperature is virtually constant and equal to $T_1 = 273$ K, with diurnal and annual variations virtually not influencing its magnitude. The velocities of the platforms on the upper boundary are known and equal to $u_c = 1$ cm/year for the continental platform and to $u_0 = 6-9$ cm/year for the oceanic platform [3, 4].

The boundary conditions for the temperature and velocities on the upper boundary of the computational domain, i.e., on the day surface of the earth, will have the following form

$$T (x , l_y , t) = T_1 ;$$

$$u \left(x < \frac{l_x}{2} , l_y , t \right) = u_c ; \quad u \left(x > \frac{l_x}{2} , l_y , t \right) = u_0 ; \quad u \left(\frac{l_x}{2} , l_y , t \right) = 0 ; \quad v (x , l_y , t) = 0 . \quad (13)$$

It is most convenient to let the lower boundary of the computational domain be at a depth at which perturbations of the temperature and velocity fields caused by subsidence of the lithospheric platform are absent. From seismic investigations of zones such as island arcs and active continental margins, it follows that a subducted platform is subsided to a depth not exceeding 700 km [3, 4]. In the adopted computational domain the lower boundary corresponds to the depth of the platform subsidence $l_y = 1000$ km. Unfortunately, there are no accurate data on the temperatures and velocities at this depth. According to the investigations and calculations carried out in [3, 4], the value of the temperature T_2 at a depth of 1000 km is taken to be equal to 2400 K and the absence of any flow is assumed for the velocity field.

Proceeding from the foregoing, the boundary conditions at the lower boundary of the region considered have the following form:

$$T(x, 0, t) = T_2; \quad u(x, 0, t) = 0; \quad v(x, 0, t) = 0. \tag{14}$$

The next complex problem is determination of the extent of the computational domain in the x direction. Since two lithospheric platforms with a total extent of 15,000–20,000 km, participate in the process of subduction (thrust over and under), then, taking into account the length of both platforms, it will be practically impossible to solve the problem, because the thickness of the subducted platform is on the order of 70 km, which is less than 1/200 of the lengths of both platforms. However, a more thorough analysis of the pattern of flow and temperature field shows that the vertical velocity and temperature profiles should change only in the zones of the formation and subsidence of the platforms [3, 4, 13]. Over the remaining portions the vertical velocity and temperature profiles do virtually not change for a rather long period of time.

Thus, if we exclude from the computational domain the portions removed from the zones of formation and subduction of the platforms at a distance of more than 2000 km, we may assume that the flow pattern and temperature fields in these zones change insignificantly. This assumption is confirmed by the results of numerical calculations.

Taking this assumption into account, we included in the computational domain two platforms, each having a length of 3000 km. It is assumed that the results of solution of the problem are quite adequate for longer platforms. Thus, the computational domain is bounded by rift zones that are characterized by periodicity conditions for temperature and by slippage conditions for velocity. The boundary conditions on the lateral boundaries of the computational domain will have the form

$$\begin{aligned} &\frac{\partial T(0, y, t)}{\partial x} = 0; \quad u(0, y, t) = 0; \quad \frac{\partial v(0, y, t)}{\partial x} = 0; \\ &\frac{\partial T(l_x, y, t)}{\partial x} = 0; \quad u(l_x, y, t) = 0; \quad \frac{\partial v(l_x, y, t)}{\partial x} = 0. \end{aligned} \tag{15}$$

Now, it is necessary to write the initial conditions. However, the selection of initial conditions for temperature is a rather complex and as yet unsolved problem. Therefore, in the present work, without claiming absolute validity of the choice, we adopt the following initial conditions:

$$\begin{aligned} &T = T_2 - \frac{T_2 - T_s}{93 \cdot 10^4} y \quad x \in [0; l_x]; \quad y \in [0; 93 \cdot 10^4]; \\ &T = T_s - \frac{T_s - T_1}{7 \cdot 10^4}(y - 93 \cdot 10^4) \quad x \in [0; l_x]; \quad y \in [0; 93 \cdot 10^4; 10^6]; \\ &u(x, y, 0) = v(x, y, 0) = 0, \end{aligned} \tag{16}$$

Here y is measured in meters.

Let us write the problem in dimensionless form. For this purpose, as a scale we take the following quantities:

the distance is given by the height of the region l_y;

$$\text{velocity by } \frac{\lambda_2}{\rho_2 c_p l_y}\text{; temperature by } (T_2 - T_1)\text{;}$$

$$\text{viscosity by } \mu_2\text{; pressure by } \frac{\lambda_2\mu_2}{\rho_2 c_p l_y^2}\text{; heat release by } \frac{\lambda_2 (T_2 - T_1)}{l_y^2}\text{; time by } \frac{l_y^2\rho_2 c_p}{\lambda_2}. \tag{17}$$

After simple transformations, the dimensionless equations that describe the natural convective heat transfer in the subduction zone will have the form

$$\frac{\rho}{\mathrm{Pr}}\left(\frac{\partial U}{\partial \tau} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y}\right) = -\frac{\partial P}{\partial X} + \frac{\partial}{\partial X}\left(2\eta\frac{\partial U}{\partial X}\right) + \frac{\partial}{\partial Y}\left(\eta\frac{\partial U}{\partial Y}\right) + \frac{\partial}{\partial Y}\left(\eta\frac{\partial V}{\partial X}\right); \tag{18}$$

$$\frac{\rho}{\mathrm{Pr}}\left(\frac{\partial V}{\partial \tau} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y}\right) = -\frac{\partial P}{\partial Y} + \frac{\partial}{\partial X}\left(\eta\frac{\partial V}{\partial X}\right) + \frac{\partial}{\partial Y}\left(2\eta\frac{\partial V}{\partial Y}\right) + \frac{\partial}{\partial X}\left(\eta\frac{\partial U}{\partial Y}\right) + \rho\mathrm{Ra}\theta; \tag{19}$$

$$\frac{\partial \rho}{\partial \tau} + \frac{\partial \rho U}{\partial X} + \frac{\partial \rho V}{\partial Y} = 0; \tag{20}$$

$$\rho C\left(\frac{\partial \theta}{\partial \tau} + U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y}\right) = \frac{\partial}{\partial X}\left(\Lambda\frac{\partial \theta}{\partial X}\right) + \frac{\partial}{\partial Y}\left(\Lambda\frac{\partial \theta}{\partial Y}\right) +$$

$$+ Q + \eta K\left[\left(\frac{\partial U}{\partial X}\right)^2 + \left(\frac{\partial V}{\partial Y}\right)^2 + \frac{1}{2}\left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X}\right)^2\right]. \tag{21}$$

Here

$$X = \frac{x}{l_y};\quad Y = \frac{y}{l_y};\quad \tau = \frac{t\lambda_2}{\rho_2 c_p l_y^2};\quad U = \frac{u\rho_2 c_p l_y}{\lambda_2};\quad V = \frac{v\rho_2 c_p l_y}{\lambda_2};\quad P = \frac{p\rho_2 c_p l_y^2}{\lambda_2\mu_2};$$

$$\rho = \frac{\rho}{\rho_2};\quad \eta = \frac{\mu}{\mu_2};\quad \Lambda = \frac{\lambda}{\lambda_2};\quad C = \frac{c_{ef}}{c_p};\quad \theta = \frac{T - T_1}{T_2 - T_1};\quad Q = \frac{q_v l_y^2}{\lambda_2 (T_2 - T_1)};$$

$$\mathrm{Pr} = \frac{c_p\mu_2}{\lambda_2} \text{ -- Prandtl number;}$$

$$\mathrm{Ra} = \mathrm{Pr}\cdot\mathrm{Gr} = \mathrm{Pr}\frac{\rho_2^2 l_y^3 g\beta (T_2 - T_1)}{\mu_2^2} \text{ -- Rayleigh number;}$$

$$K = \frac{2\mu_2\lambda_2}{\rho_2^2 c_p^2 l_y^2 (T_2 - T_1)} \text{ -- dimensionless number.}$$

The boundary conditions in dimensionless form are

$$\theta(X, 1, \tau) = 0;\quad \theta(X, 0, \tau) = 1; \tag{22}$$

$$\frac{\partial \theta(0, Y, \tau)}{\partial X} = \frac{\partial \theta(l_x/l_y, Y, \tau)}{\partial X} = 0; \tag{23}$$

$$U\,(X < l_x/2l_y,\ 1,\ \tau) = U_c = u_c\,\frac{\rho_2 c_p l_y}{\lambda_2}\,; \tag{24}$$

$$U\,(X > l_x/2l_y,\ 1,\ \tau) = U_0 = u_0\,\frac{\rho_2 c_p l_y}{\lambda_2}\,; \tag{25}$$

$$U\,(0,\ Y,\ \tau) = U\,(l_x/l_y,\ Y,\ \tau) = U\,(X,\ 0,\ \tau) = 0\,; \tag{26}$$

$$\frac{\partial V\,(0,\ Y,\ \tau)}{\partial X} = \frac{\partial V\,(l_x/l_y,\ Y,\ \tau)}{\partial X} = V\,(X,\ 0,\ \tau) = 0\,; \tag{27}$$

$$C = \begin{cases} 1 & \text{for } \theta < \theta_s\,; \\ 1 - \text{sign}\,(\theta^{\tau + \Delta\tau} - \theta^{\tau})\,\dfrac{1}{\text{Ste}}\,\dfrac{\partial\psi}{\partial\theta} & \text{for } \theta_s \le \theta \le \theta_l\,; \\ 1 & \text{for } \theta > \theta_l\,; \end{cases}$$

$$\frac{\partial\psi}{\partial\theta} = \frac{6\,(\theta - \theta_s)\,(\theta - \theta_l)}{(\theta_l - \theta_s)^3}\,;$$

Here $\text{Ste} = c_p(T_2 - T_1)/L_F$ is the Stefan number.

The initial conditions in dimensionless form are

$$\theta = 1 - \frac{1 - \theta_s}{0.93}\,Y \quad X \in [0,\ l_x/l_y]\,; \quad Y \in [0,\ 0.93]\,; \tag{28}$$

$$\theta = \theta_s\,\frac{1 - Y}{0.07} \quad X \in [0,\ l_x/l_y]\,; \quad Y \in [0.93,\ 1]\,; \tag{29}$$

$$U\,(X,\ Y,\ 0) = V\,(X,\ Y,\ 0) = 0\,.$$

Estimating the inertial terms on the left sides of equations of motion (18) and (19) and taking into consideration the value of the Prandtl number $\text{Pr} = 2.4 \cdot 10^{22}$, we conclude that they are much smaller than the viscosity terms; therefore, we ignore them in what follows.

The values of the dimensionless numbers for the problem are: $\text{Ra} = 1.7 \cdot 10^7$; $\text{Pr} = 2.4 \cdot 10^{22}$; $\text{Ste} = 6.4$; $K = 3 \cdot 10^{-8}$.

Gravitational convection in a liquid layer appears when $\text{Ra} \ge \text{Ra}_{cr}$, where Ra_{cr} is the critical Rayleigh number, whose value depends on the type of boundary conditions. For the problem considered $\text{Ra}_{cr} \sim 1700$, which is much smaller than $\text{Ra} = 1.7 \cdot 10^7$. From this we can conclude that the convection is developed in the upper mantle f the earth, since the Pr number is large; the intensity of convection in the mantle is insignificant: the motion of the mantle is slow and the mode of flow is laminar.

It is evident that the dimensionless complex K, which takes into account the contribution of dissipation energy to the energy balance, will be important only in the region of high velocity gradients.

The differential equations and boundary conditions (18)-(29) fully describe a mathematical model that allows one to calculate the geometry of the melting front of the oceanic lithosphere in the zone of subduction.

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